

ATOM-PHOTON BOUND STATES

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Arrays of compact superconducting resonators in which we have embedded two artificial atoms. Study of the atom-field interaction in the single- and double-excitation subspaces. Coherent interaction between two atom-photon bound states (APBS) in the resonant and dispersive regimes: implementation of SWAP and C π 2-qubit gates.

Novel circuit QED implementation of photonic lattices coupled to quantum emitters: arrays of Josephson junction resonators and transmon qubits.

↳ geometry facilitates the arrangement of time-domain and frequency control of the qubits: exploitation of individual APBS properties.

architecture: 21 high-impedance resonators and 2 transmon qubits

↳ i) mode structure of the array

ii) emergence of APBS, mixing of single and double excitations

iii) numerics in the double-excitation manifold

iv) coherent population exchange between two APBS

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Structure of the paper

- 1) sample and experimental setup
- 2) transmission spectroscopy
 - i) bare coupled cavity array
 - ii) dressed coupled cavity array and APBS
- 3) state preparation
 - i) single-excitation subspace
 - ii) double-excitation subspace
- 4) APBS interaction
 - i) 2-APBS level splitting
 - ii) time resolved excitation exchange (SWAP)
 - iii) 2-APBS π interaction (CZ)
- 5) conclusion

SAMPLE AND EXPERIMENTAL SETUP

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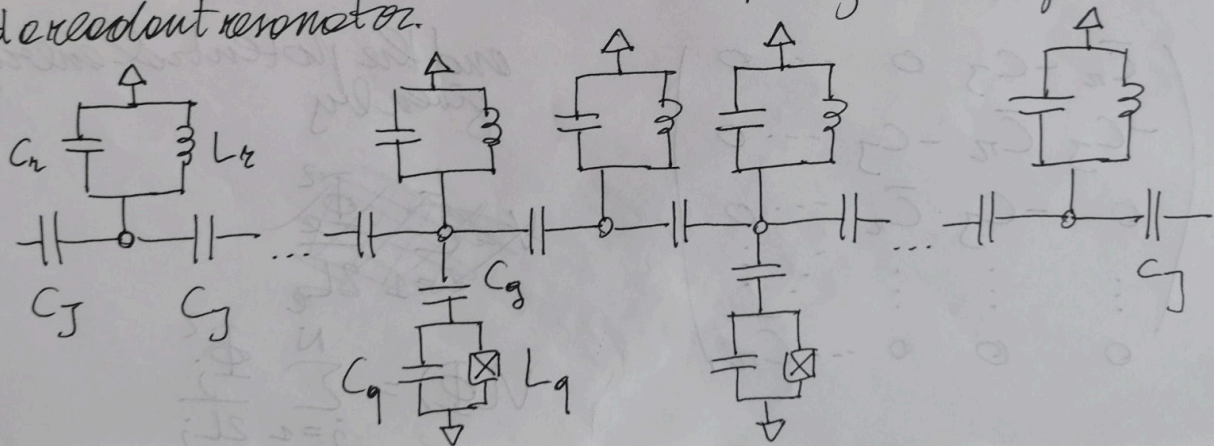
We have a transmission line made out of 21 high-impedance microwave resonators forming a coupled-cavity array.

resonator: array of 10 Josephson junctions, with $L_J = 8.87 \text{ mH}$, $C_J = 81.3 \text{ pF}$

$\rightarrow \omega_J = 5.583 \text{ GHz}$ and $Z_J = 312 \Omega$

edge resonators coupled to a 50Ω coplanar waveguide.

Artificial atoms: superconducting flux-tunable transmons capacitatively coupled to the sites 10 and 12 of the array. Coupling to a charge line, a flux line and a readout resonator.



TRANSMISSION SPECTROSCOPY

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To characterize the bare coupled cavity array, we tune the resonant frequencies of the qubits far away. We now study the transmission spectroscopy.

Circuit quantization procedure:

$$\text{Lagrangian: } \mathcal{L} = \frac{1}{2} \dot{\vec{\Phi}}^T \mathcal{C} \dot{\vec{\Phi}} - V(\vec{\Phi})$$

we can write the capacitance of each resonator as $\bar{C}_n = C_n + 2C_J$ if $C_J \ll C_n$. In this way, the capacitance matrix reads

$$\mathcal{C} = \begin{pmatrix} \bar{C}_n - C_J & 0 & \dots & 0 \\ -C_J & \bar{C}_n - C_J & \dots & 0 \\ 0 & -C_J & \bar{C}_n & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \bar{C}_n \end{pmatrix}$$

and the potential energy is given by

$$V(\vec{\Phi}) = \sum_{j=1}^N \frac{\Phi_j^2}{2L_j}$$

The Hamiltonian of the system can be obtained with the Legendre^s transformation which reads

$$H = \vec{Q}^T \dot{\vec{\Phi}} - L \quad \text{where} \quad \vec{Q} = \frac{\partial L}{\partial \dot{\vec{\Phi}}} = C \dot{\vec{\Phi}}$$

$$= \frac{1}{2} \vec{Q}^T C^{-1} \vec{Q} + V(\vec{\Phi}) = H(\vec{Q}, \vec{\Phi})$$

What is now C^{-1} ? In general, the inverse of a tridiagonal matrix is a dense matrix, but we know that $\bar{C}_n \gg C_J$ and we can estimate C^{-1} by means of perturbation theory as:

$$C^{-1} \simeq \frac{1}{\bar{C}_n} \begin{pmatrix} 1 & C_J/\bar{C}_n & -(C_J/\bar{C}_n)^2 & \dots \\ C_J/\bar{C}_n & 1 & C_J/\bar{C}_n & \dots \\ -(C_J/\bar{C}_n)^2 & C_J/\bar{C}_n & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \simeq \begin{pmatrix} 1/\bar{C}_n & C_J/\bar{C}_n^{-2} & 0 & \dots \\ C_J/\bar{C}_n^2 & 1/\bar{C}_n & \dots & \dots \\ 0 & C_J/\bar{C}_n^2 & \dots & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

we discarded the terms involving $1/\bar{C}_n^k$, $k \geq 3$ because of the condition $\bar{C}_n \gg C_J$. Now we quantize H and we write

$$\hat{Q}_j = \sqrt{\frac{C_n \omega_n}{2}} (\hat{e}_j^+ + \hat{e}_j) \quad \hat{\Phi}_j = i \sqrt{\frac{1}{2 C_n \omega_n}} (\hat{e}_j^+ - \hat{e}_j)$$

which satisfy $[\hat{\Phi}_j, \hat{Q}_{j'}] = i \delta_{jj'}$. The quantum Hamiltonian finally reads as

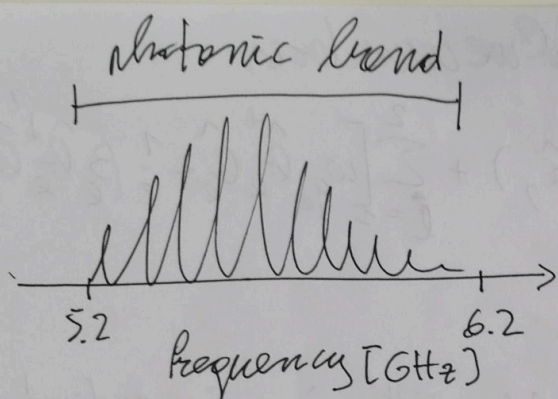
$$\hat{H} = \omega_n \sum_{j=1}^N \hat{e}_j^+ \hat{e}_j + J \sum_{j=1}^{N-1} (\hat{e}_{j+1}^+ \hat{e}_j + \hat{e}_j^+ \hat{e}_{j+1}) \quad J = \frac{C_J \omega_n}{2 C_n}$$

Where $[\hat{e}_j, \hat{e}_{j'}^+] = \delta_{jj'}$ and open boundary conditions are imposed. The above Hamiltonian is quadratic and can be analytically diagonalized. If we introduce the eigenmodes of the system

$$\hat{e}_k = \sqrt{\frac{2}{N+1}} \sum_{j=1}^N \sin(k_j) \hat{e}_j \quad \text{the Hamiltonian reduces to}$$

$$\hat{H} = \sum_k \omega_k \hat{e}_k^+ \hat{e}_k, \quad \omega_k = \omega_n + 2J \cos(k) \quad k = \frac{m\pi}{N+1} \quad m = 1, \dots, N$$

↳ collection of decoupled harmonic oscillators of frequencies $\omega_k \in [\omega_n - 2J, \omega_n + 2J]$



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$$\rightarrow J/2\pi = 250 \text{ MHz}$$

presence of disorder $S\omega_z/2\pi = 257 \text{ MHz}$

but $S\omega_z/J = 1/10$

at higher photon numbers, the Kerr nonlinearity emerges ($K/2\pi = 100 \text{ MHz}$)

Now we add the transmon qubits to the description of the system. We start by adding one qubit to the coupled cavity array and we tune its frequency across the photonic band. The transmission coefficient is affected by the presence of the qubit. There is a minimum in the transmission in correspondence with the bare qubit frequency (the incoming field is reflected back). Each mode presents a definite standing wave spatial profile, which sets its effective interaction strength with the artificial atom.

As the atom frequency approaches the band, it hybridizes with band-edge photons. According to its frequency, the bound state has a mainly atomic

or mainly photonic character. The model we consider is

$$\hat{H}_{\text{free}} = \sum_{j=1}^N \omega_j \hat{a}_j^\dagger \hat{a}_j + \sum_{j=1}^{N-1} J (\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{j+1}^\dagger \hat{a}_j) + \sum_{\ell=1}^2 \left[\omega_\ell \hat{b}_\ell^\dagger \hat{b}_\ell + \frac{1}{2} \beta_\ell \hat{b}_\ell^\dagger \hat{a}_\ell^2 \right] \\ + \sum_{\ell=1}^2 g_\ell (\hat{a}_j^\dagger \hat{b}_\ell + \hat{b}_\ell^\dagger \hat{a}_j)$$

We now study the physics of the above Hamiltonian, in order to unveil the APBS it describes. We consider one qubit and we restrict to the single-excitation subspace. We have

$$\hat{H}_{\text{red}} = 2J \sum_k \cos(k) \hat{a}_k^\dagger \hat{a}_k + \delta \hat{a}_k^\dagger \hat{a}_k + g \sqrt{\frac{2}{N+1}} \sum_k \sin(k \times q) (\hat{a}_k^\dagger \hat{b}_1 + \hat{a}_k^\dagger \hat{b}_2) \\ \hookrightarrow \delta = \omega_q - \omega_r$$

The APBS is an eigenstate of the total Hamiltonian whose corresponding energy lies outside the photonic band, so we need to solve

$$\hat{H}_{\text{red}} |\Phi\rangle = \omega |\Phi\rangle \quad \text{with } |\omega| > 2J \quad \text{since we are within the single-excitation subspace,}$$

the eigensate is a linear combination of the single-excitation basis

$$|\Phi\rangle = \left(c_q \hat{b}^\dagger + \sum_k u_k \hat{a}_k^\dagger \right) |g, 0\rangle \quad \text{and by inserting this Ansatz in the eigenvalue equation we get}$$

$$\omega - \delta = \Sigma(\omega) \quad \text{with } \Sigma(\omega) \text{ self-energy given by}$$

$$\Sigma(\omega) = \frac{2g^2}{N+1} \sum_k \frac{\sin^2(kx_q/2)}{\omega - 2J \cos(k/2)} = \frac{g^2}{2J} \frac{1 - e^{-\frac{2x_q}{\lambda}} - e^{-\frac{2(N+1-x_q)}{\lambda}}}{\sinh\left[\frac{1}{\lambda} \left(1 - e^{-2\frac{N+1}{\lambda}}\right)\right]}$$

$$\text{with } \lambda(\omega) = \left[\text{arccosh} \frac{|\omega|}{2J} \right]^{-1}$$

the solutions with $|\omega| > 2J$ correspond to the ABS. If $\omega \rightarrow \pm 2J$ then the condition is

$$g^2 > \frac{J(N+1)(2J \mp \delta)}{x_q(N+1 - x_q)} \quad \text{and in the thermodynamic limit}$$

$$\Sigma(\omega) \approx \frac{g^2}{\omega \sqrt{1 - 4J^2/\omega^2}} \quad \text{and we get the equation}$$

$$\omega_{BS} - \omega_q = \frac{g^2}{(\omega_{BS} - \omega_c) \sqrt{1 - \frac{g^2}{(\omega_{BS} - \omega_c)^2}}}$$

equation for the energy¹⁰
of the APBS

The ground state can be worked out as

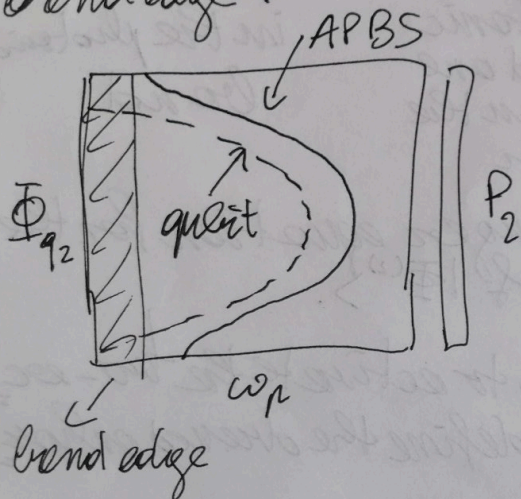
$$|\Phi\rangle = [\cos\theta \hat{a}^{\dagger} + (-1)^s \sin\theta \hat{\alpha}^{\dagger}] |g, 0\rangle$$

where $\hat{\alpha} = \sum_x f(x) \hat{a}_x$ and s is the sign of ω_{BS} (above or below the photonic band). The mixing angle measures the degree of hybridization. The function $f(x)$ contains the exponential $e^{-\frac{|x-x_q|}{\lambda}}$ which indicates how the motional mode is exponentially localized around the atom position x_q .

STATE PREPARATION

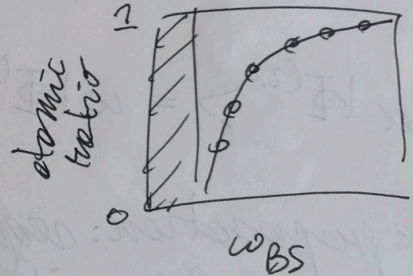
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We consider a localized atom-photon bound state. To excite the APBS, we send a microwave pulse (π -pulse on Q_2) and we read out the qubit via the measurement resonator. We measure the population P_2 . The APBS frequency differs from the qubit frequency and approaches the photonic band edge:



and from this measurement one can extract the mixing angle θ , since the reduced drive on the qubit is

$$\Omega_R = \Omega_{R0} \cos \theta$$



We now go in the double-excitation subspace, where two particles can be exchanged between the qubit and the photonic band. Now the

nonlinear nature of the transmon starts to play a role. Let's start¹² by writing down the Ansatz for the ZAPBS:

$$|\Phi^{(2)}\rangle = \left[\frac{1}{\sqrt{2}} \sum_{j,k=1}^2 c_{jk} \hat{b}_j^+ \hat{b}_k^+ + \sum_{j=2}^2 \sum_x c_j(x) \hat{b}_j^+ \hat{e}_x^+ + \frac{1}{\sqrt{2}} \sum_{x,y} u(x,y) \hat{e}_x^+ \hat{e}_y^+ \right] |g,0\rangle$$

$\swarrow$
 one excitation in
the transmon j and
one excitation in the
transmon k

$\downarrow$
 one excitation
in the photonic
band and one
excitation in the
transmon

$\downarrow$
 two excitations
in the photonic
band

$$\hat{H}_{\text{full}} |\Phi^{(2)}\rangle = \omega |\Phi^{(2)}\rangle \rightarrow \text{Schrödinger equation for the existence of } |\Phi^{(2)}\rangle.$$

Gate preparation: sequence of two pulses to activate the two-excitation manifolds, with frequencies $\omega_{1,2}$. We define the driven on-harmonicity:

$$\Delta_{\text{driven}} = \omega_{BS}^{(2)} - \omega_{BS}^{(1)} \rightarrow \text{number of excitations} \Rightarrow \text{well below the level qudit onharmonicity}$$

APBS INTERACTION

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We now investigate the interaction between bound states. Joint resonant of the two qubits. Avoided crossing in the excitation spectrum.

↳ 2 APBS interact: hybridization and superposition of even and odd states

Let's develop the theory. The equation for 2 APBS reads

$$(\omega - \delta_1 - \Sigma_1)(\omega - \delta_2 - \Sigma_2) = \Sigma_1 \Sigma_2 e^{-\frac{2}{\lambda} |x_1 - x_2|}$$

being $\delta_j = \omega_{q_j} - \omega_R$. ↳ up to 4 possible solutions

2 APBS in the upper part done the photonic analog. We get

$$|\Phi_{\pm}\rangle = \frac{1}{\sqrt{1+|\xi|^2}} [\hat{D}_{\pm}^{\dagger}(x_1) \pm \xi \hat{D}_{\pm}^{\dagger}(x_2)] |g_1, g_2, 0\rangle$$

$$\hat{D}_{\pm}(x_j) = \underbrace{\cos(\vartheta_{\pm})}_{\text{mixing angle}} \hat{b}_j + \sin(\vartheta_{\pm}) \frac{1}{N_{\pm}} \sum_x e^{-\frac{|x-x_j|}{\lambda_{\pm}}} \hat{a}_x$$

↳ normalization factor

ζ quantifies the amount of hybridization:

$$\zeta(\omega_{\pm}) = \frac{\sqrt{\Sigma_1 \Sigma_2} e^{-\frac{|x_1 - x_2|}{\lambda}}}{\omega_{\pm} - \delta_2 - \Sigma_2}$$

Question: what happens if the qubits are far detuned from the band edge? so that $\omega_{q_j} - \omega_2 - 2J \gg g_j$ and $\cos \theta_{\pm} \approx 1$

Bruegger-Wadert-perturbation theory: eliminate the band and we get the effective Hamiltonian

$$\hat{H}_{\text{eff}} = \frac{g_1 g_2}{\delta_e} \underbrace{\frac{2}{\lambda} e^{-\frac{|x_j - x_u|}{\lambda}} \left(\hat{b}_j^\dagger \hat{b}_u + \hat{b}_u^\dagger \hat{b}_j \right)}_{\text{coherent excitation transfer between the two qubits}} + \dots$$

coherent excitation transfer between the two qubits

→ tunable 2-qubit gate!